Washington University in St. Louis
JAMES MCKELVEY SCHOOL OF ENGINEERING

TOPOLOGICAL SIMPLIFICATION OF NESTED SHAPES

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SGP 2022

Nested Shapes

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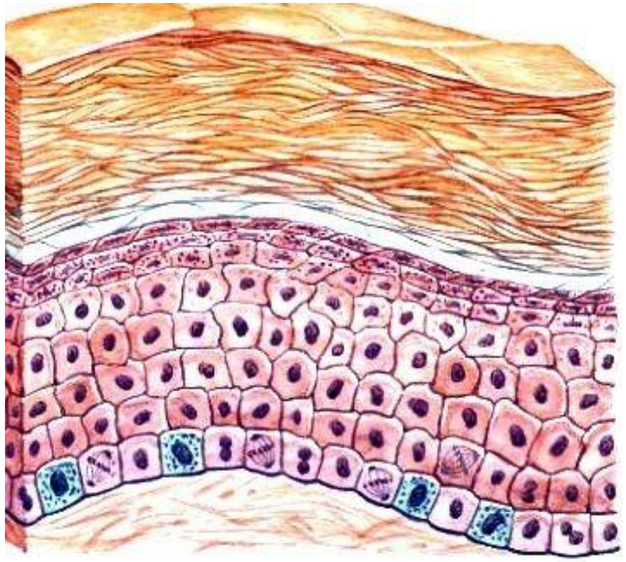
- A sequence of monotonically expanding shapes

Wikipedia

Nested Shapes

- Multi-layered structures

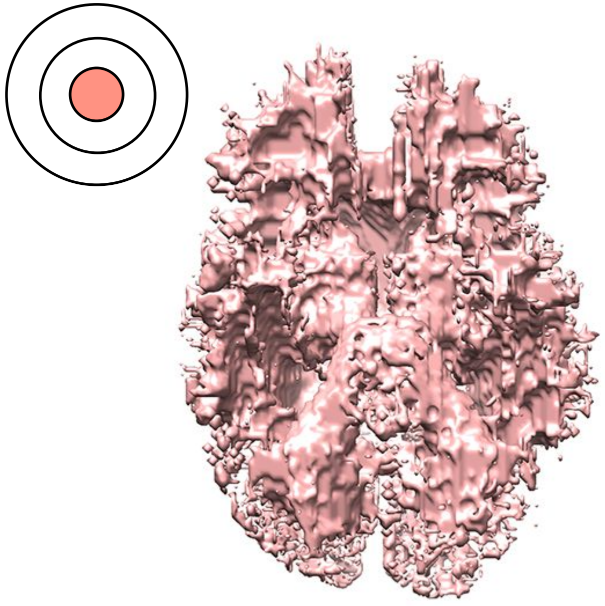
Geological strata



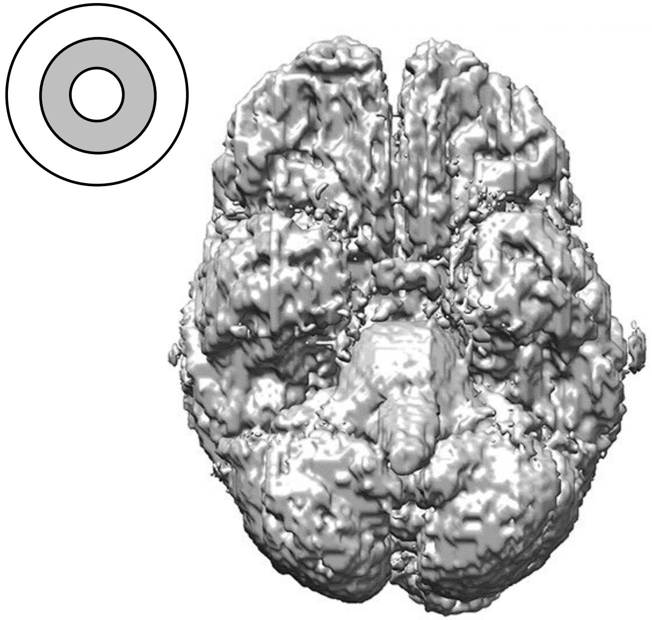
Tissue layers

Nested Shapes

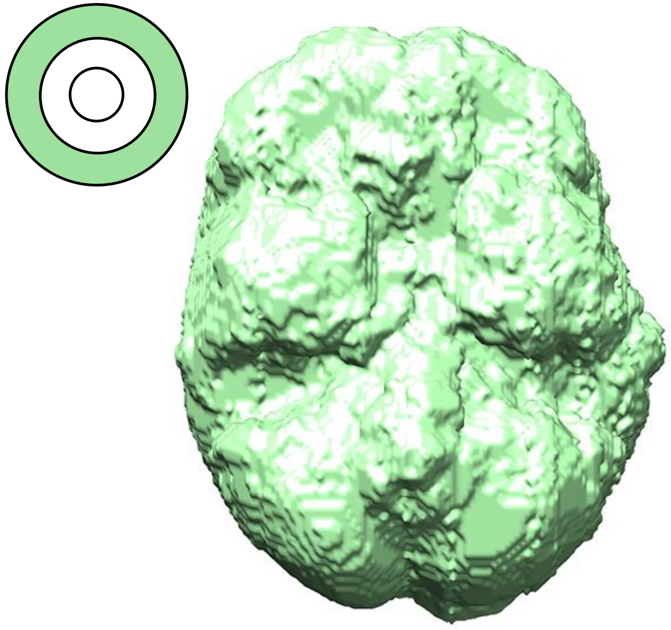
- Multi-layered structures



Cerebrospinal fluid



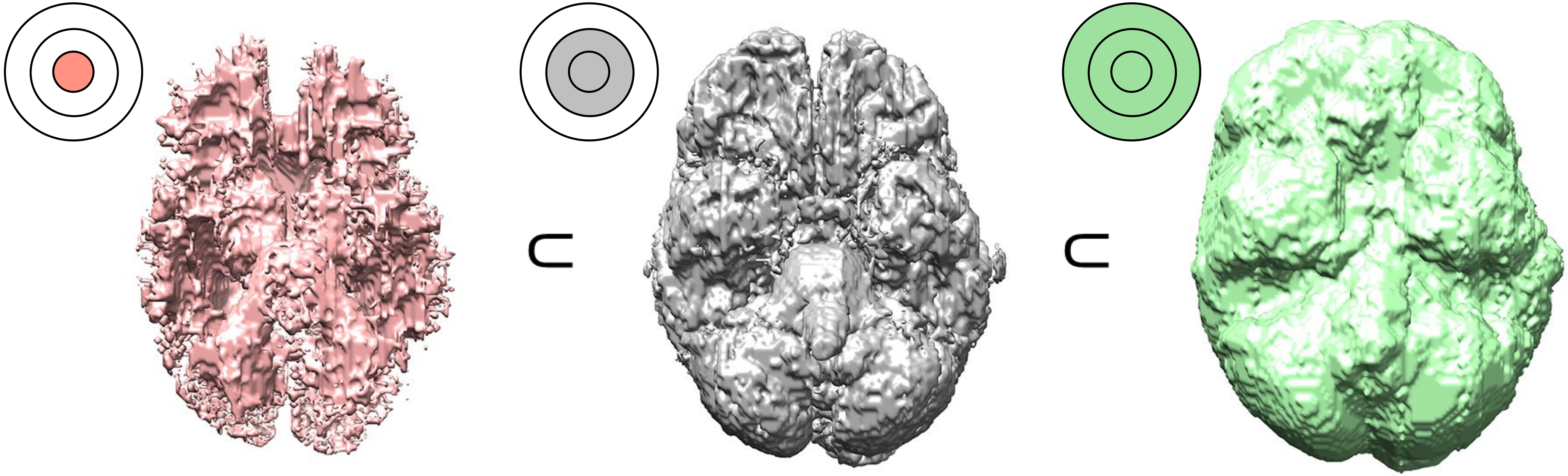
White matter



Gray matter

Nested Shapes

- Multi-layered structures
  - The outer surface of each layer forms a nested sequence



Cerebrospinal fluid ⊂ White matter ⊂ Gray matter

Nested Shapes

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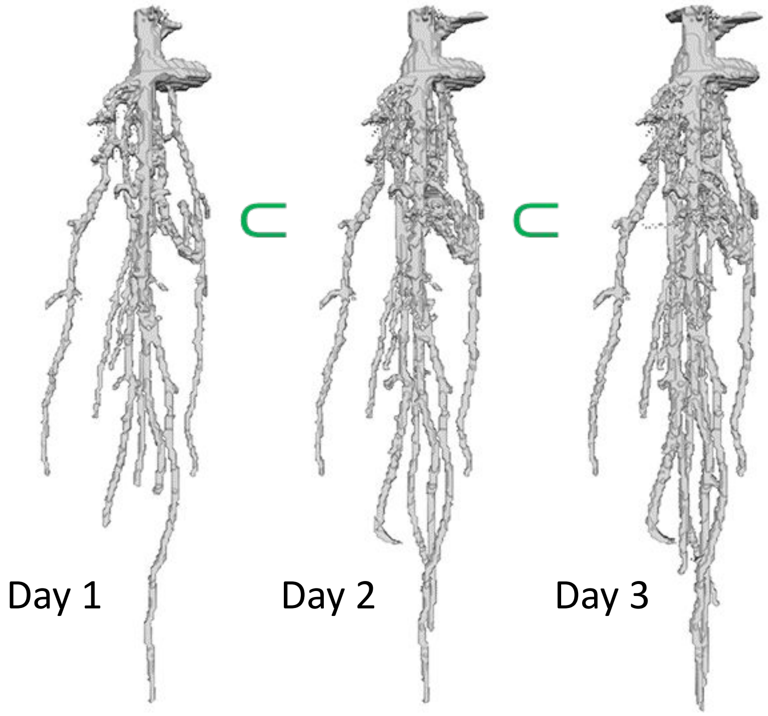
- Growing plant roots

Wikipedia

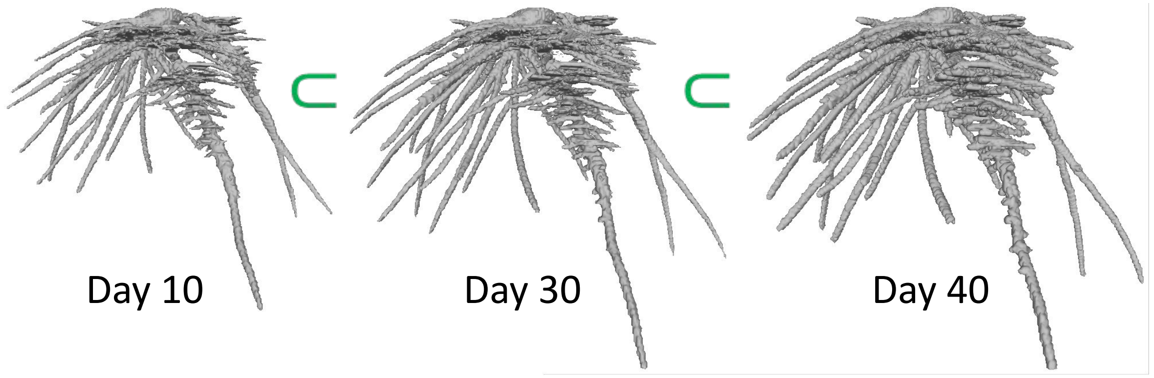
Nested Shapes

- Growing plant roots

Rice roots



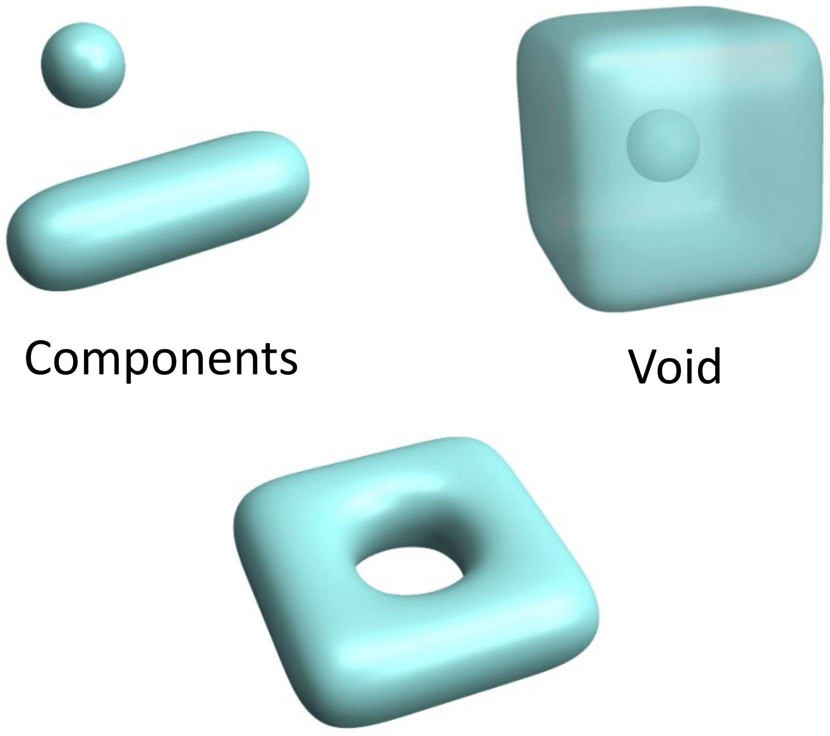
Maize (corn) roots



Topology

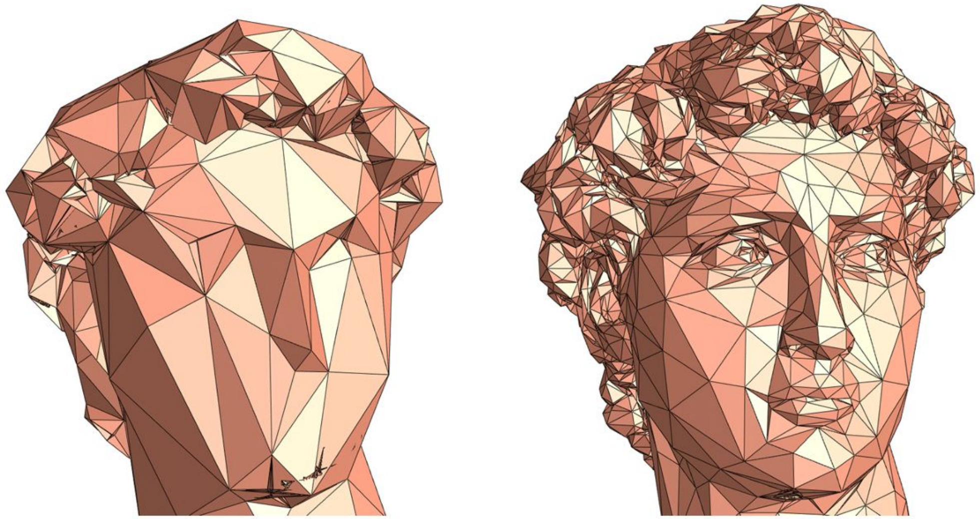
- Invariant to continuous geometric deformation

Wikipedia



Topology

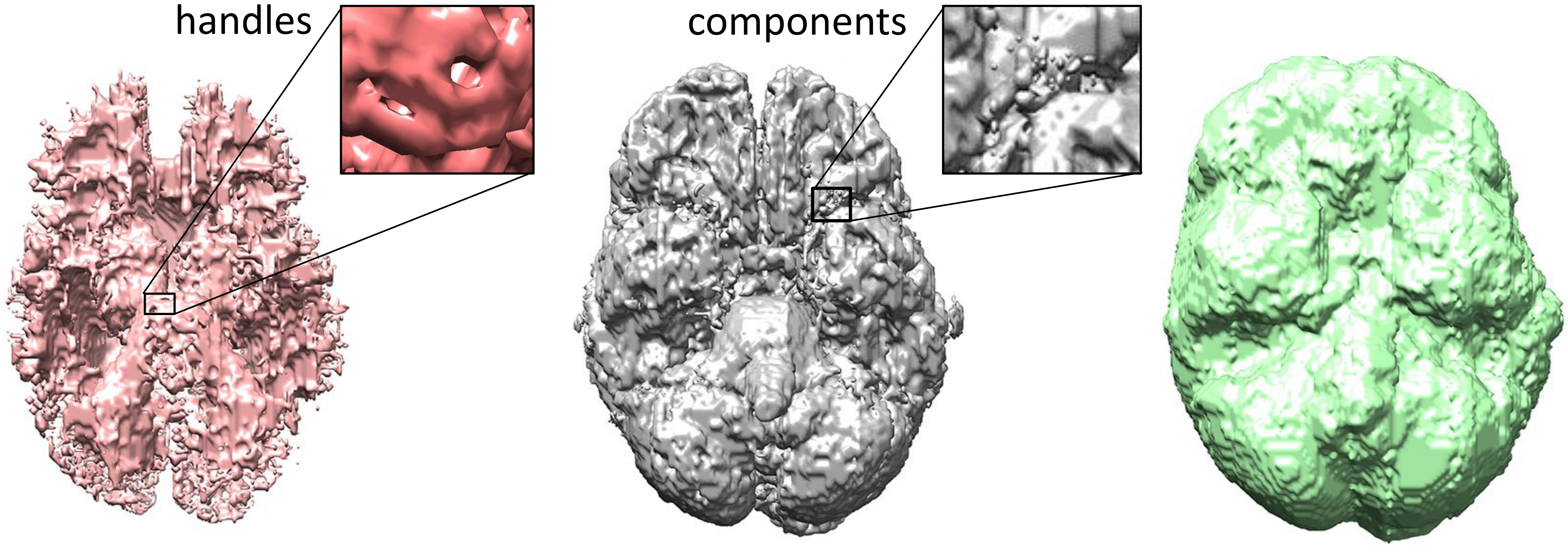
- Invariant to continuous geometric deformation
- Many geometry processing tasks are sensitive to topology:
  - Mesh simplification and fairing
  - Surface parameterization
  - Geodesic distances
  - Surface matching
  - Physical simulations



Mesh simplification before and after removing redundant topological features [Wood 04]

Topological Errors

- Reconstruction may introduce unwanted topological features



Cerebrospinal fluid White matter Gray matter

Topological Errors

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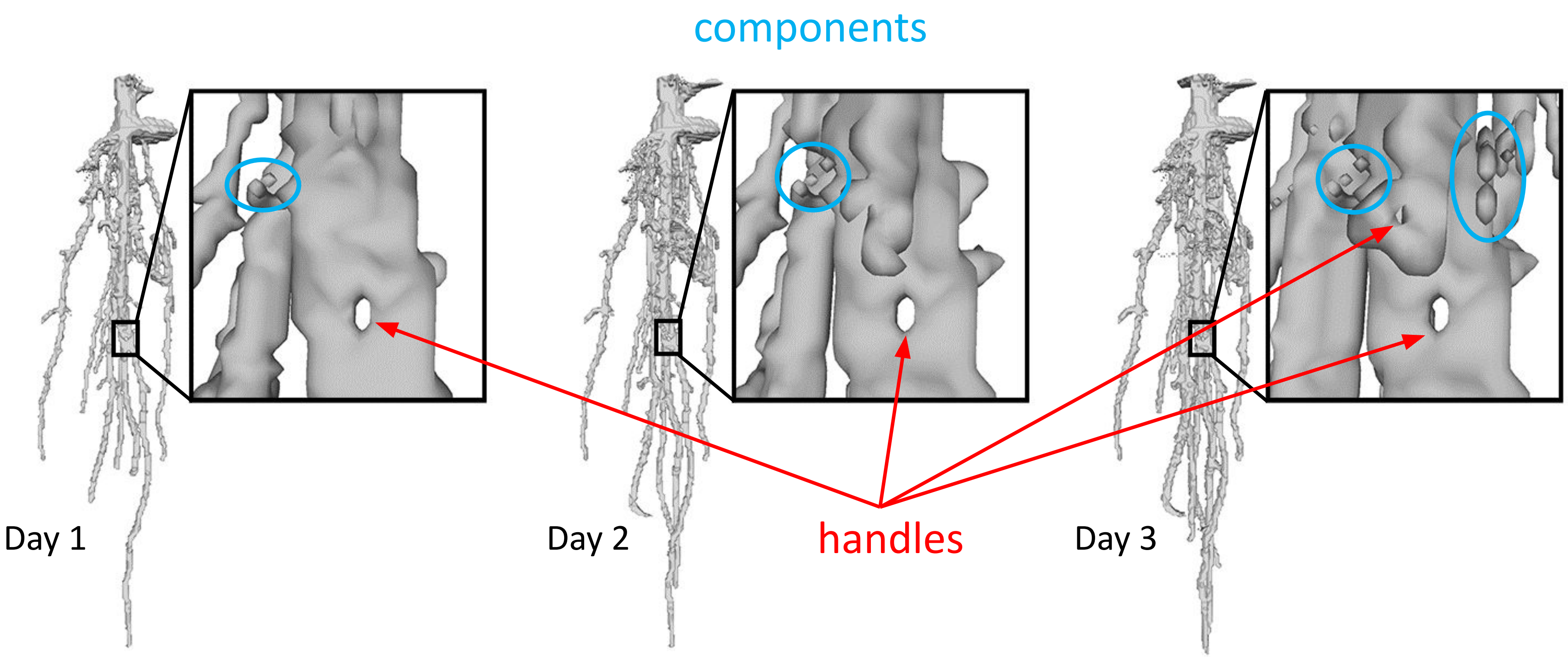
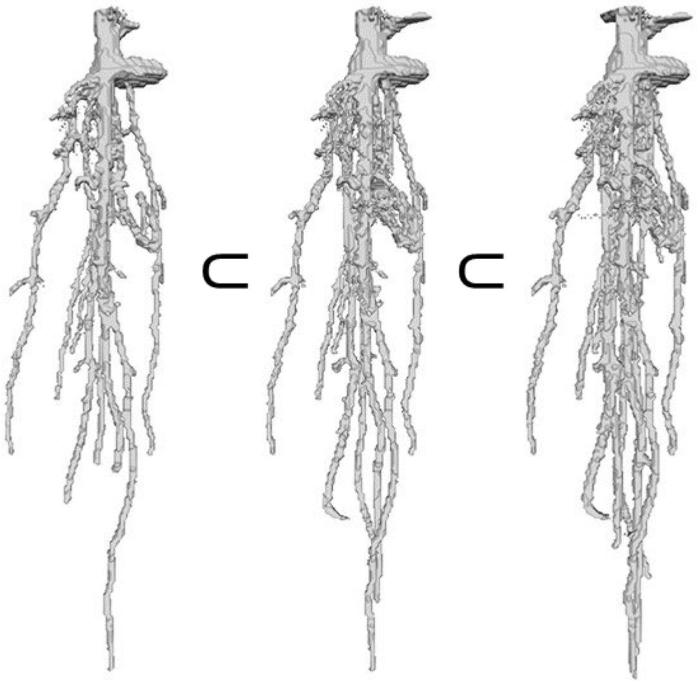
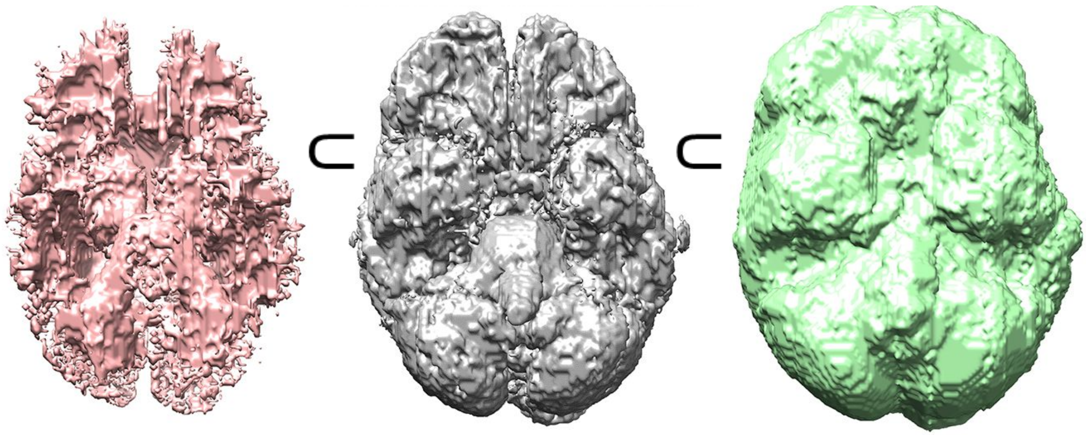
- Reconstruction may introduce unwanted topological features



Goal

- Remove unwanted topological features in reconstructed shapes
  - Maintain nesting (necessary for defining layers or modeling root growth)





Previous Works

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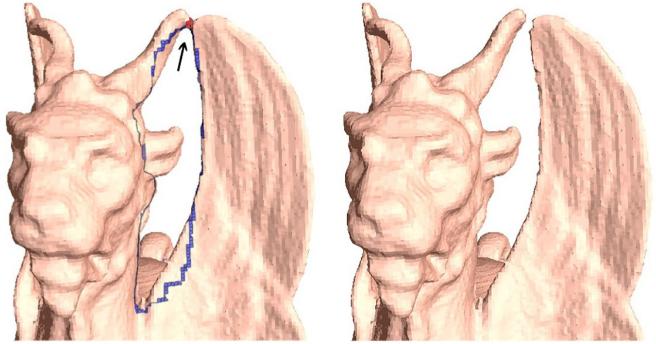
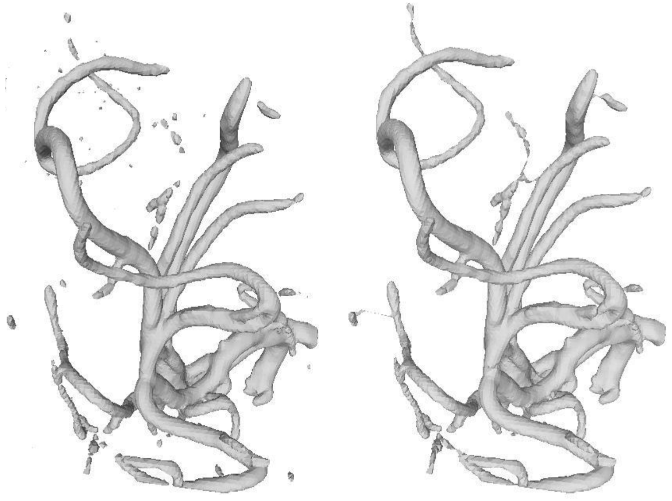
- Simplifying the topology of one shape
  - Removing handles [Shattuck 01; Han 02; Wood 04; Chen 06; Zhou 07; Segonne 07]
  - Removing all features
    - Morphological opening/closing [Nooruddin 03]
    - Inflation and deflation [Kriegeskorte 01; Bischoff 02; Szymczak 03]
    - Local heuristics [Ju 07]
    - Global optimization [Zeng 20]



[Wood 04]



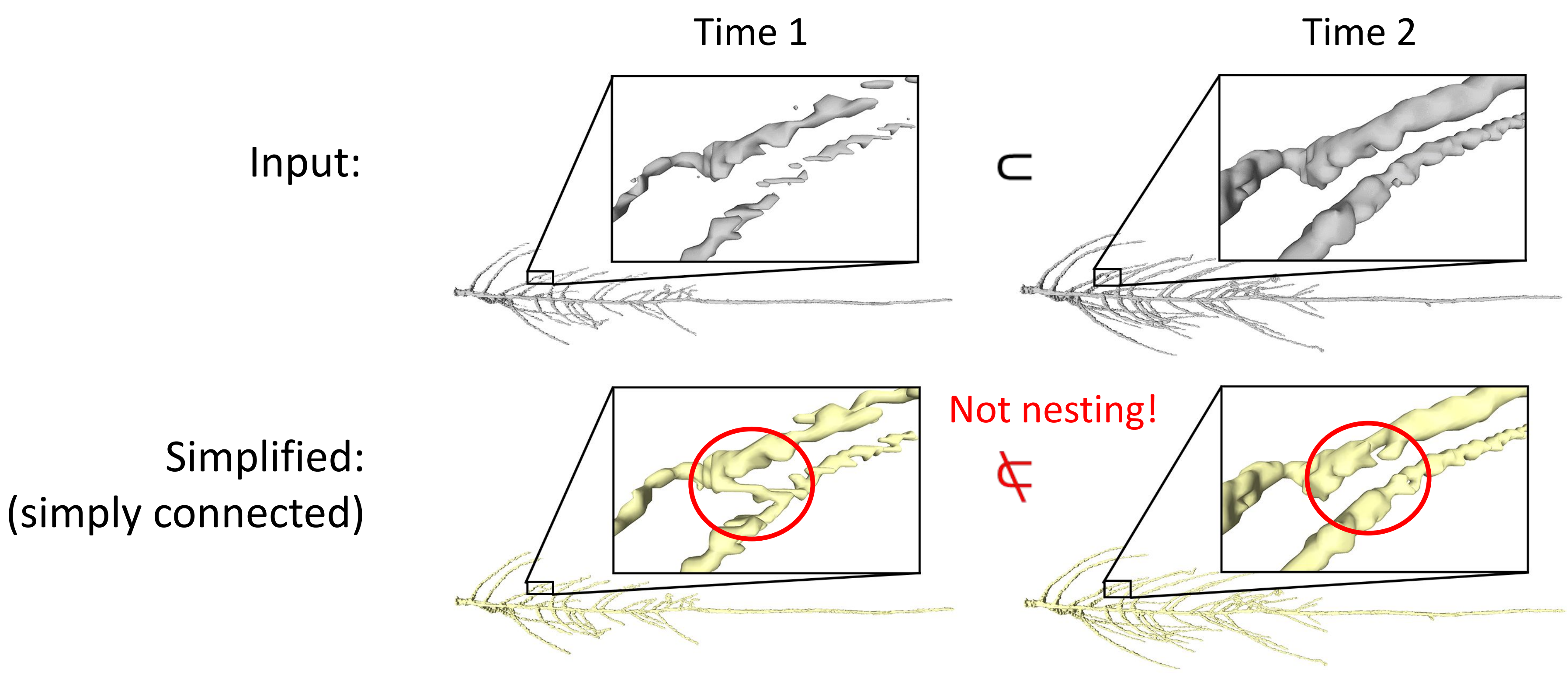
Input

Simply connected

[Zeng 20]

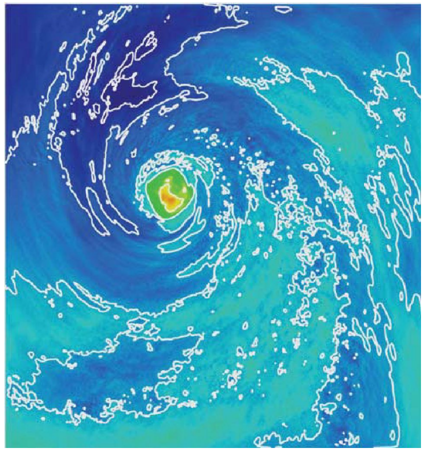
Previous Works

- Simplifying the topology of one shape
  - Cannot guarantee nesting when applied independently to each shape



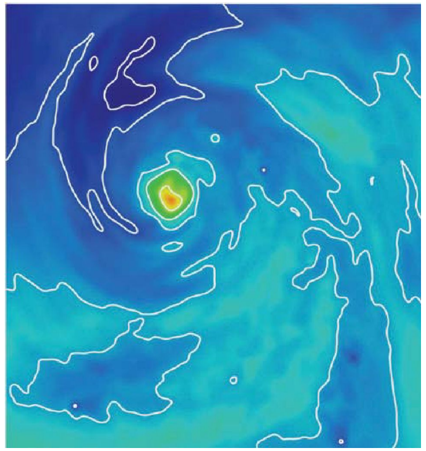
Previous Works

- Simplifying the topology of a scalar function
  - Removes extraneous critical points, thus simplifying the topology of *all* level sets (which are nested)
  - Numerical optimization [Bremer 04; Patane 09; Weinkauf 10; Gunther 14]
  - Combinatorial methods [Edelsbrunner 06; Bauer 12; Tierny 12,17; Lukasczyk 20]



14492 extrema

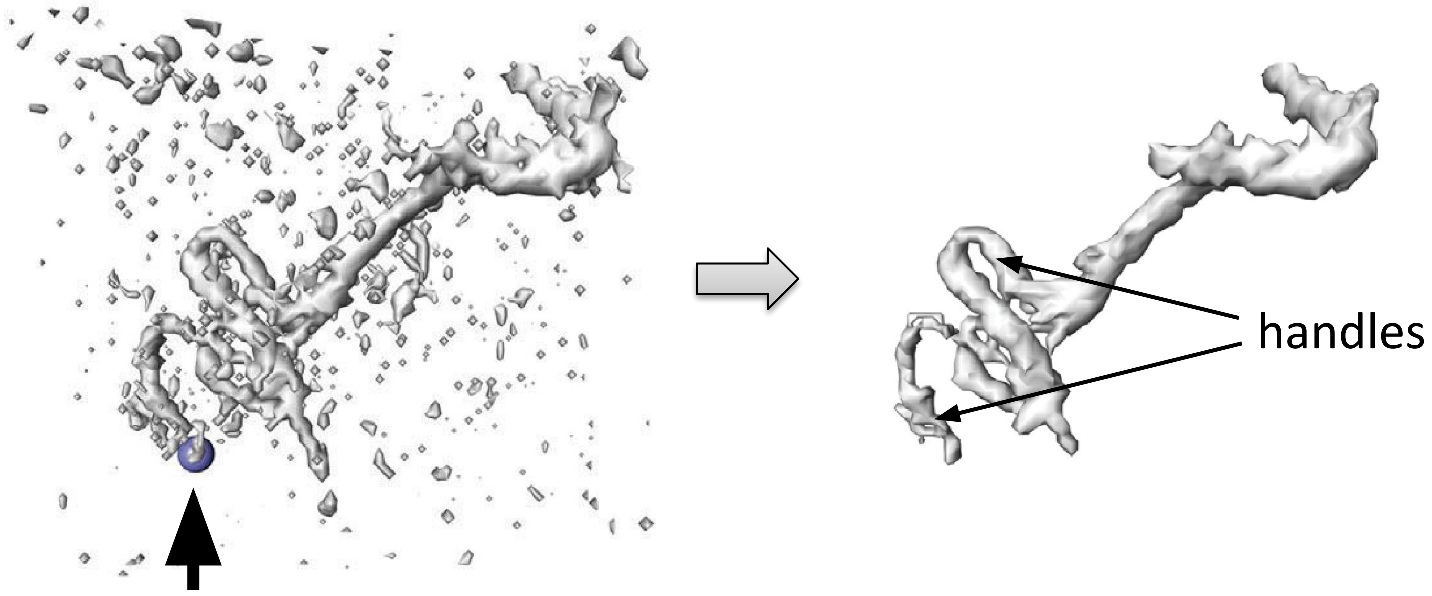
[Gunther 14]



12 extrema

Previous Works

- Simplifying the topology of a scalar function
  - Saddles in 3D (corresponding to handles of the level sets) are challenging to remove



Removing all local minima except one [Gunther 14]

Our Work

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- Simplifies the topology of a shape sequence while maintaining nesting
  - Removes all three types of topological features (components, handles, voids)
  - Minimally alters the shapes

- Technical contributions
  - Extension of the single-shape method of [Zeng 20]
  - Formulation as a discrete optimization problem
  - An efficient and effective solver

Topological Operators

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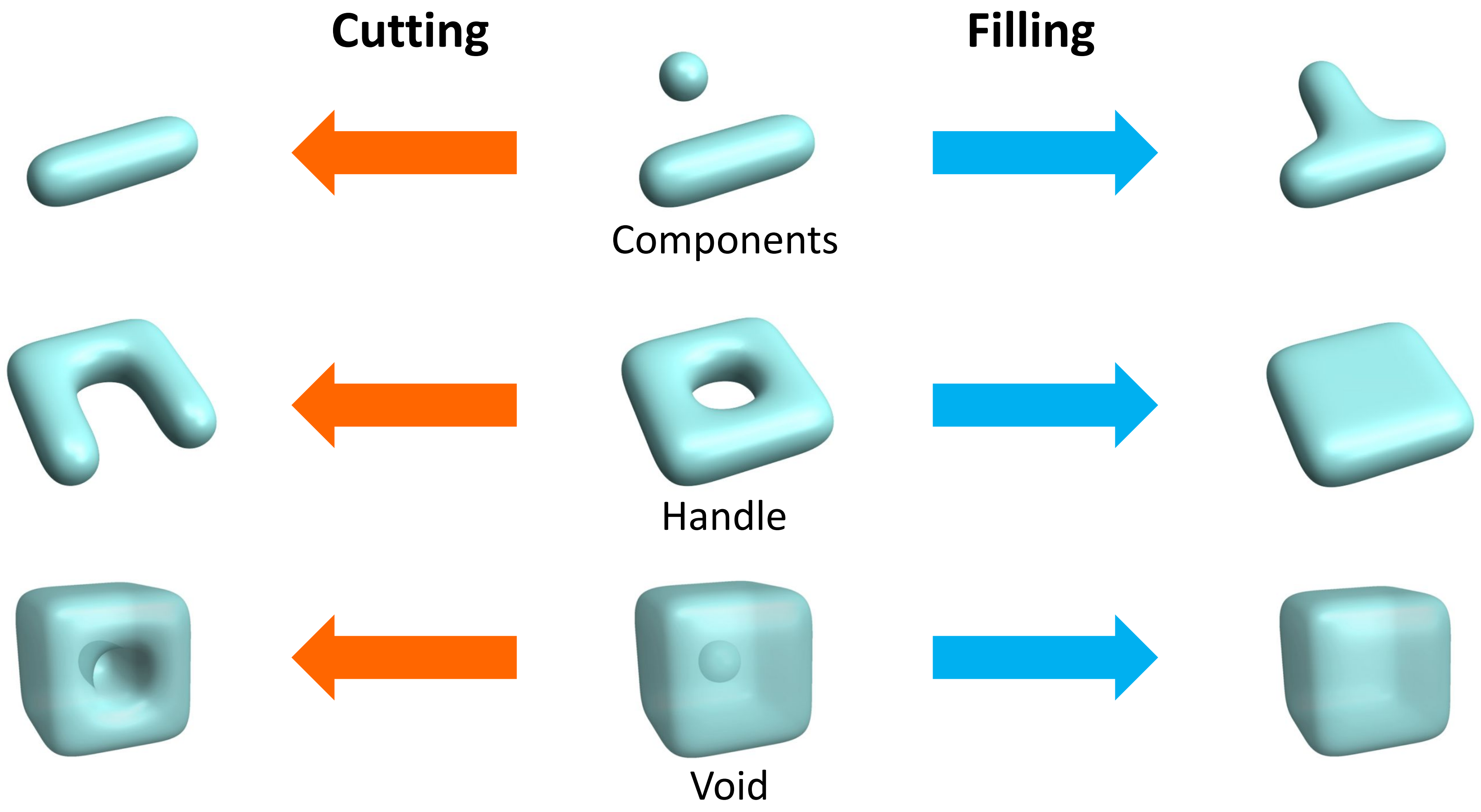
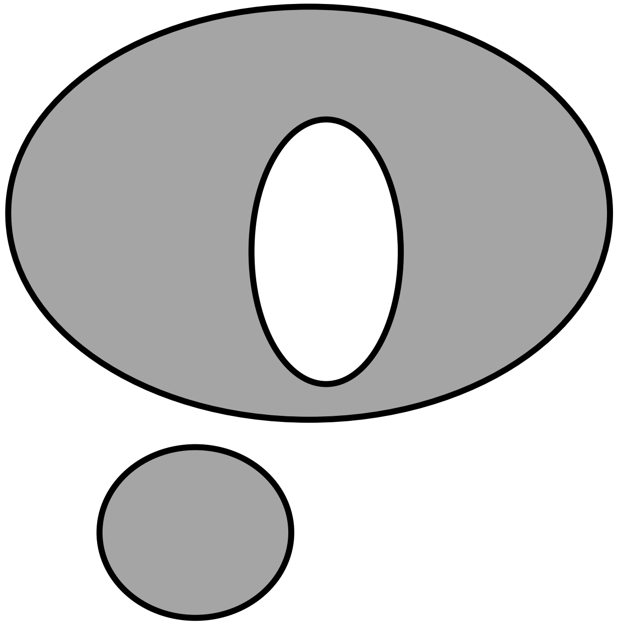
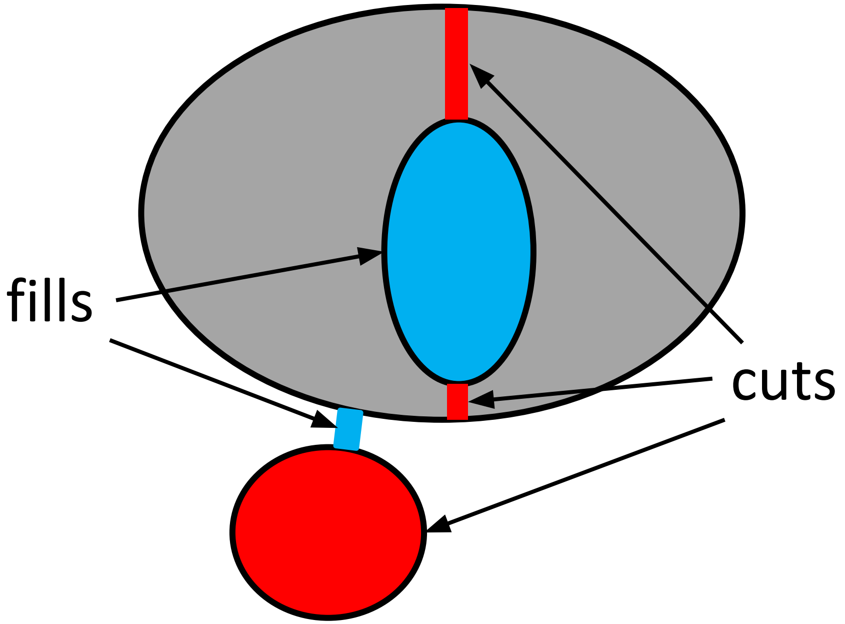


Single-shape Simplification [Zeng 20]



Single-shape Simplification [Zeng 20]

- Compute candidate cuts and fills
  - Applying a cut or fill removes one or more features
  - Each candidate associated with a cost
- Select a subset of candidates that:
  - Maximally removes topological features
  - Minimizes total cost



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- Compute candidate cuts and fills
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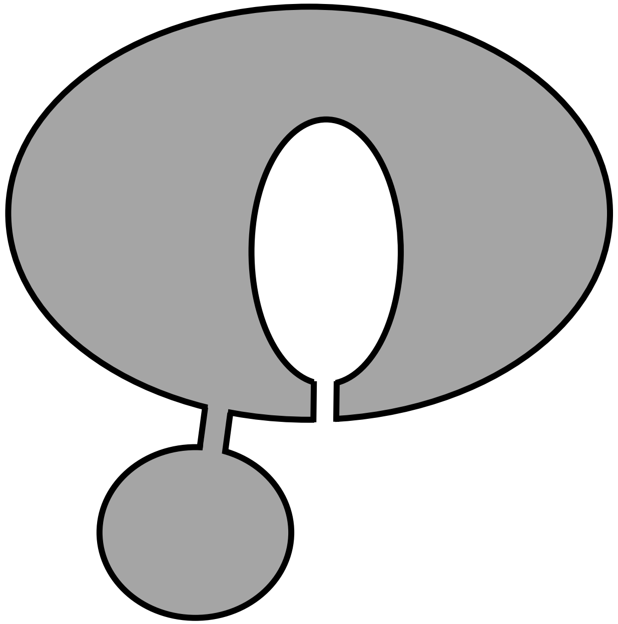
  - Minimizes total cost
- Solved as a graph labelling problem



Nesting-Aware Candidates

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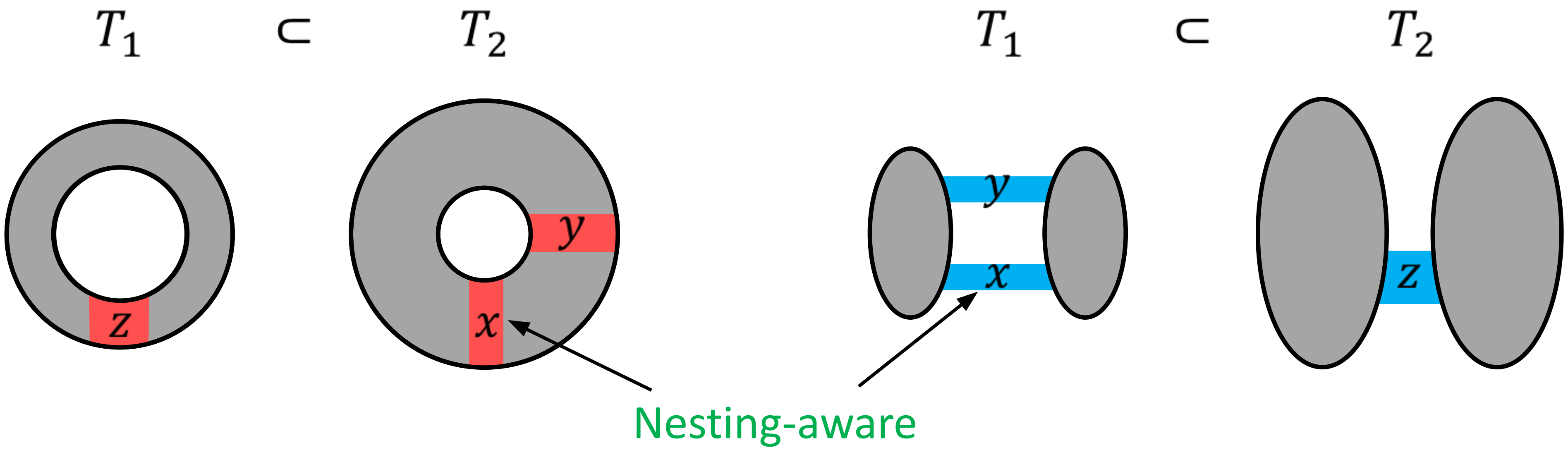

- Cut x may be used (if z is also used)
- Cut y may never be used

- Fill x may be used (if z is also used)
- Fill y may never be used

Optimization Problem

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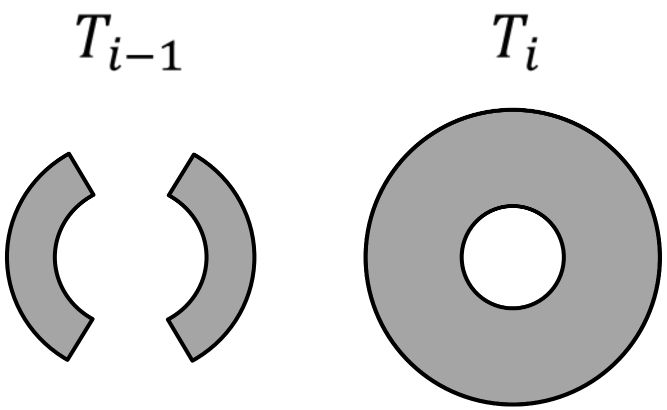
- Given:
  - Nested shapes $\{T_1 \subset \cdots \subset T_n\}$
  - Nesting-aware candidates $\{X_1, \ldots, X_n\}$, each with a cost
- Label candidates as inside (1) or outside (0) to:
  - Maximally remove topological features of each shape
  - Minimize total costs of 0-labelled cuts and 1-labelled fills
  - Maintain nesting

[Zeng 20]



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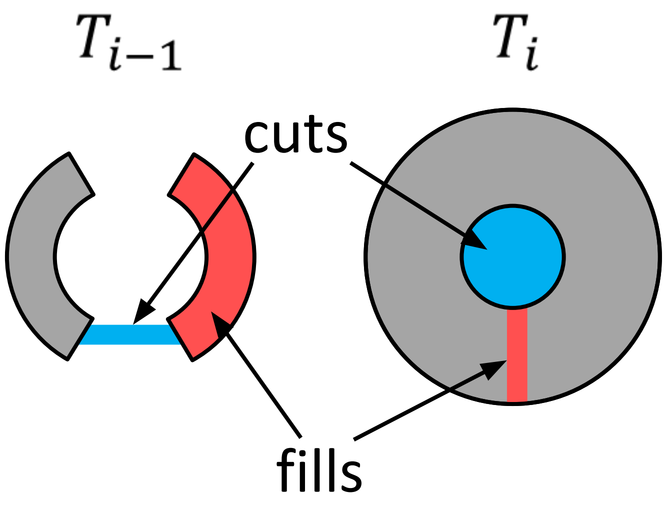
[Zeng 20]



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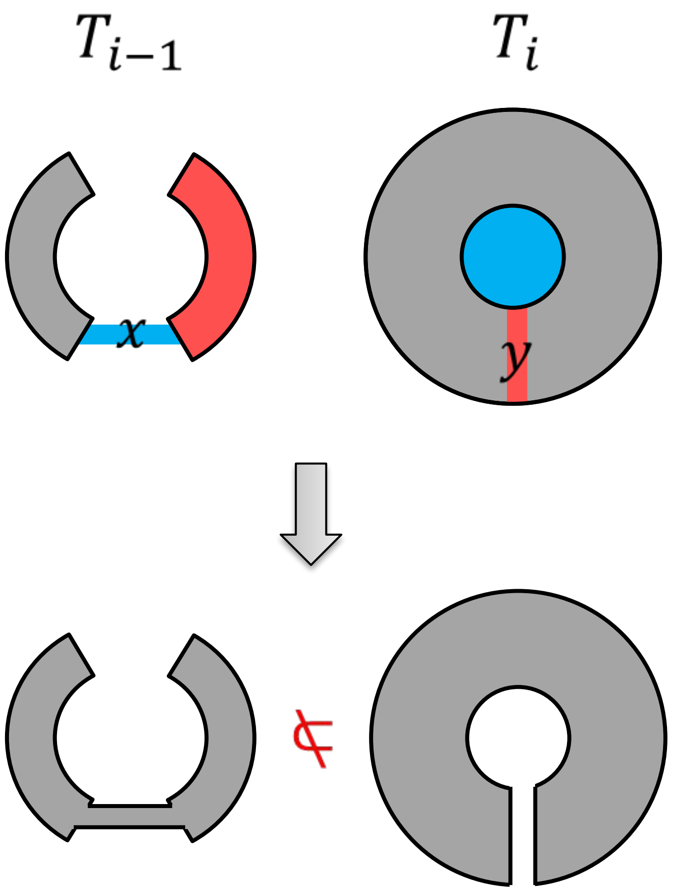

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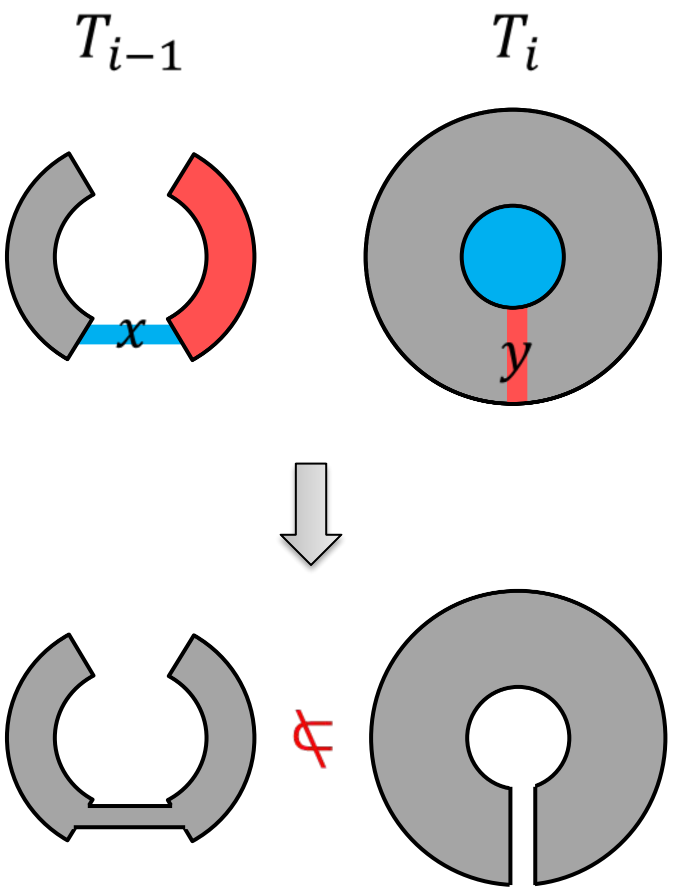
  [Zeng 20]
  - Avoid conflicting labels
    - Conflict: $x \in X_{i-1}$ overlaps with $y \in X_i$, x has label 1, y has label 0



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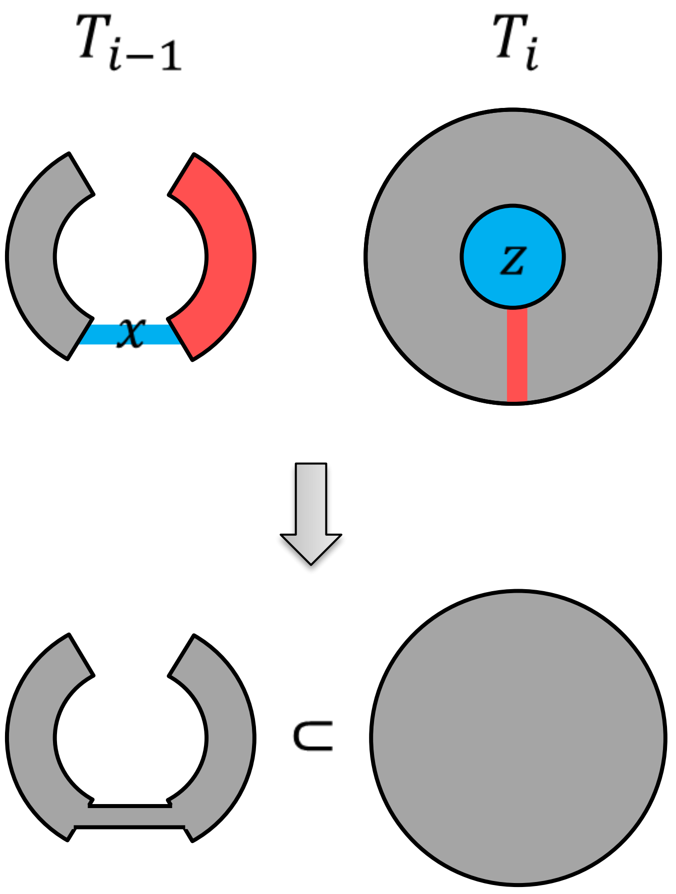
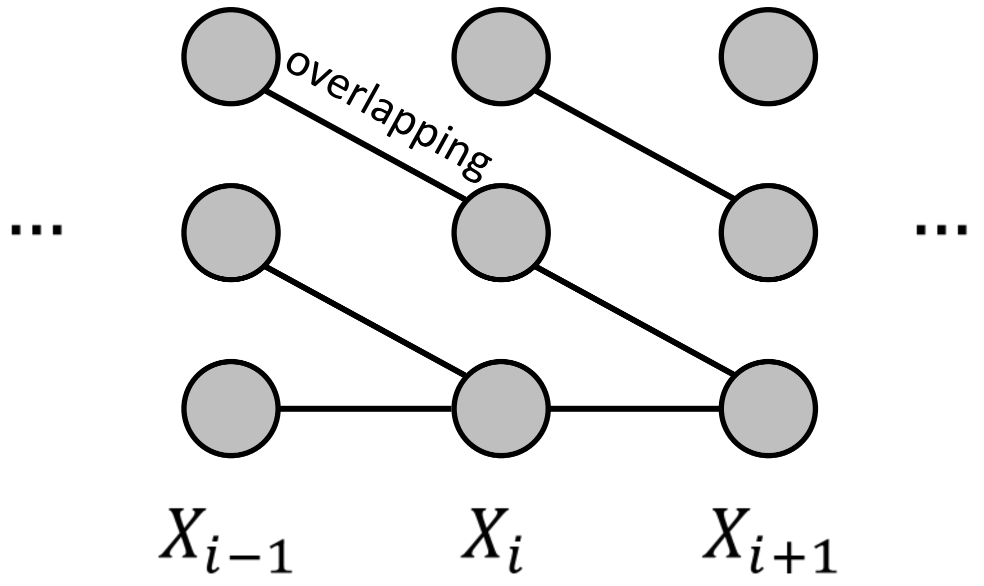
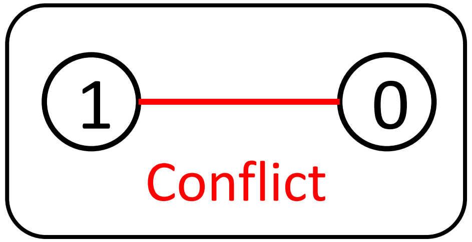


Solver 1: Label Propagation

- Propagate labels from one shape to others while avoiding conflicts
  - Use [Zeng 20] to optimize labels on each shape





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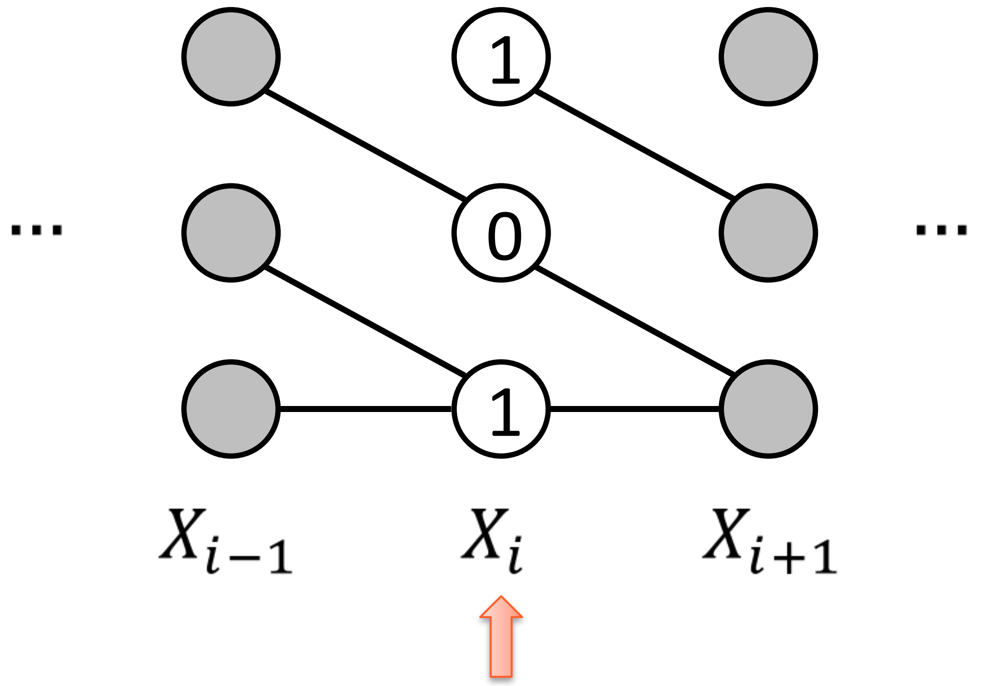

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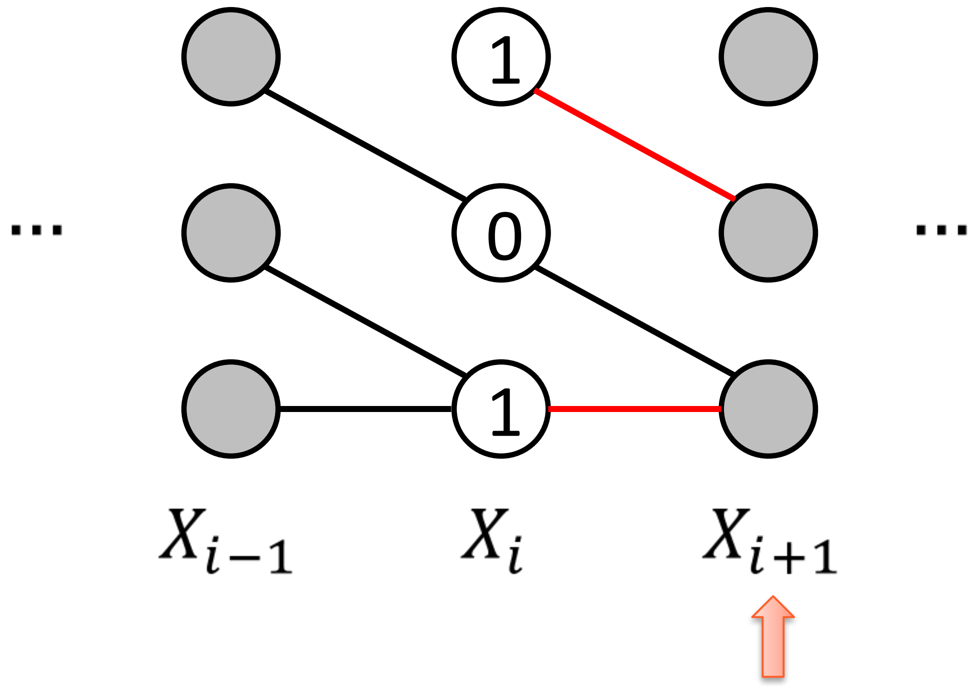

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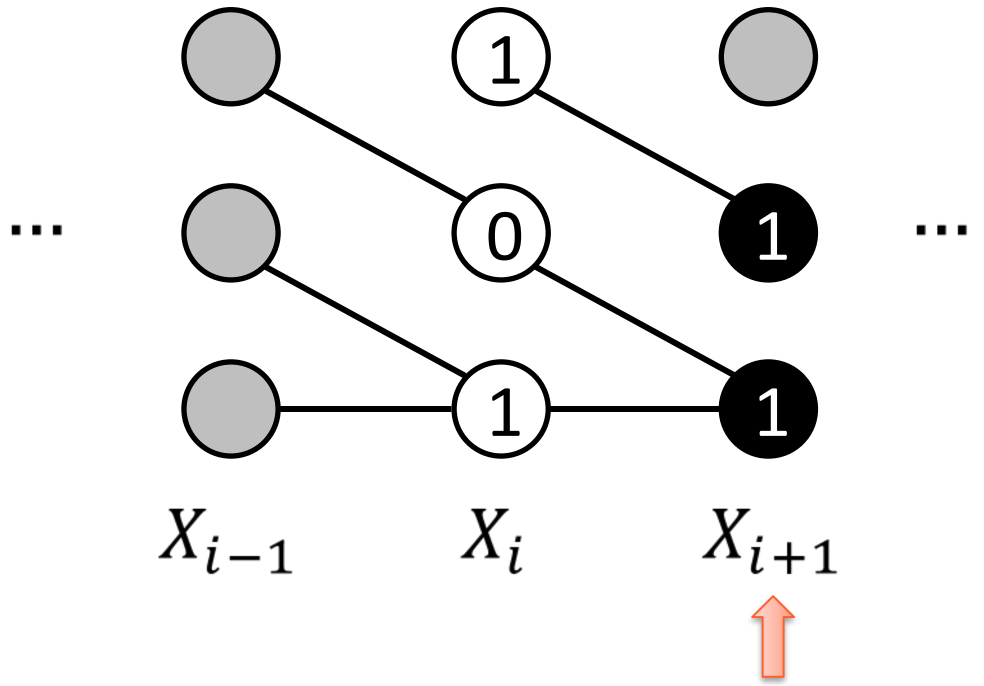

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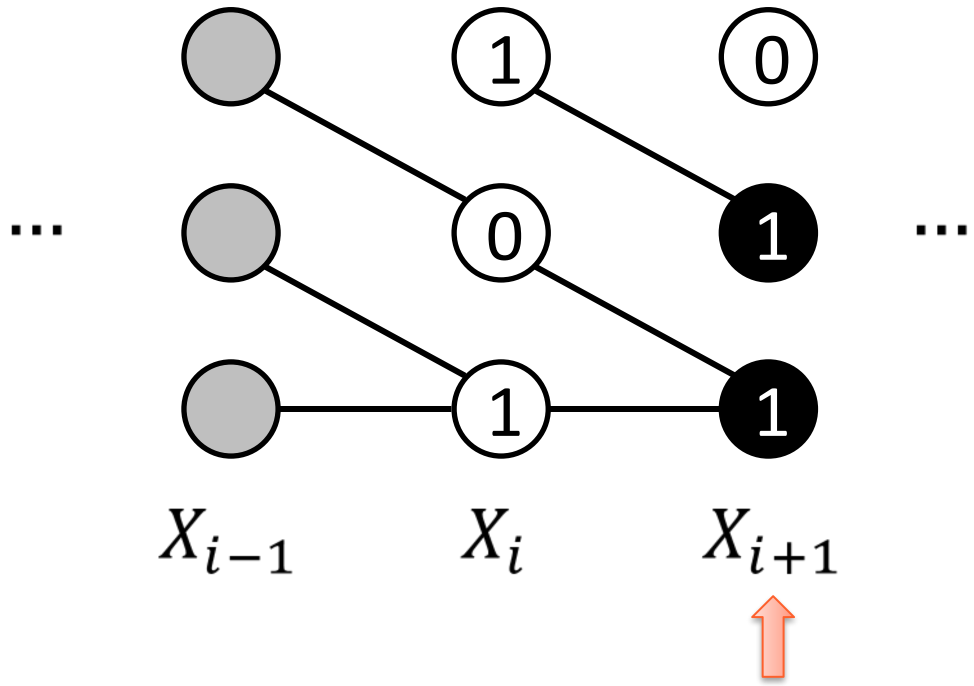

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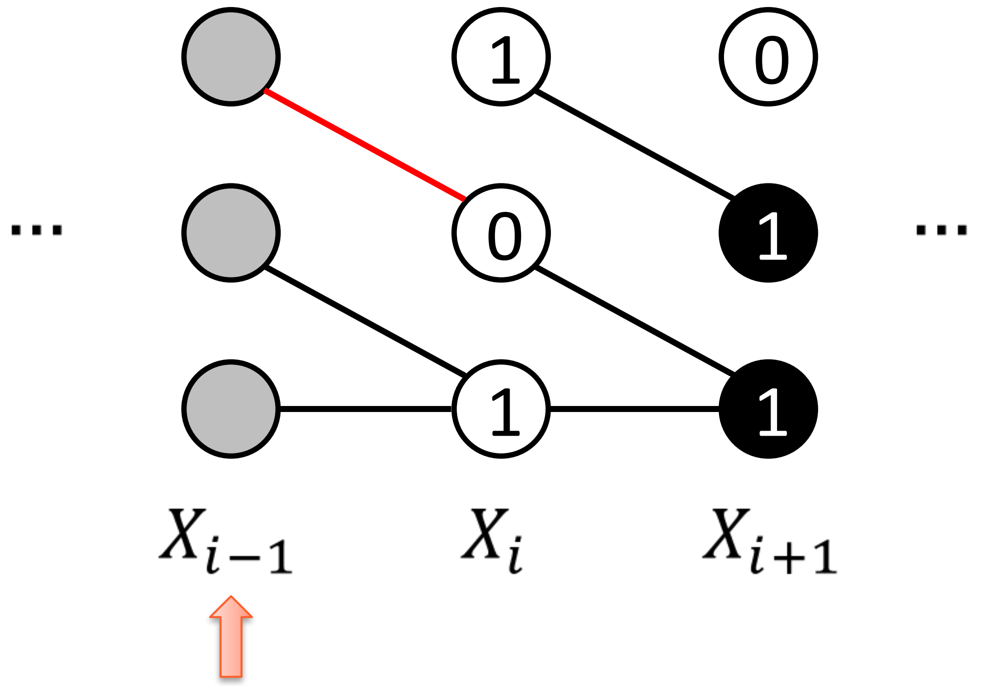

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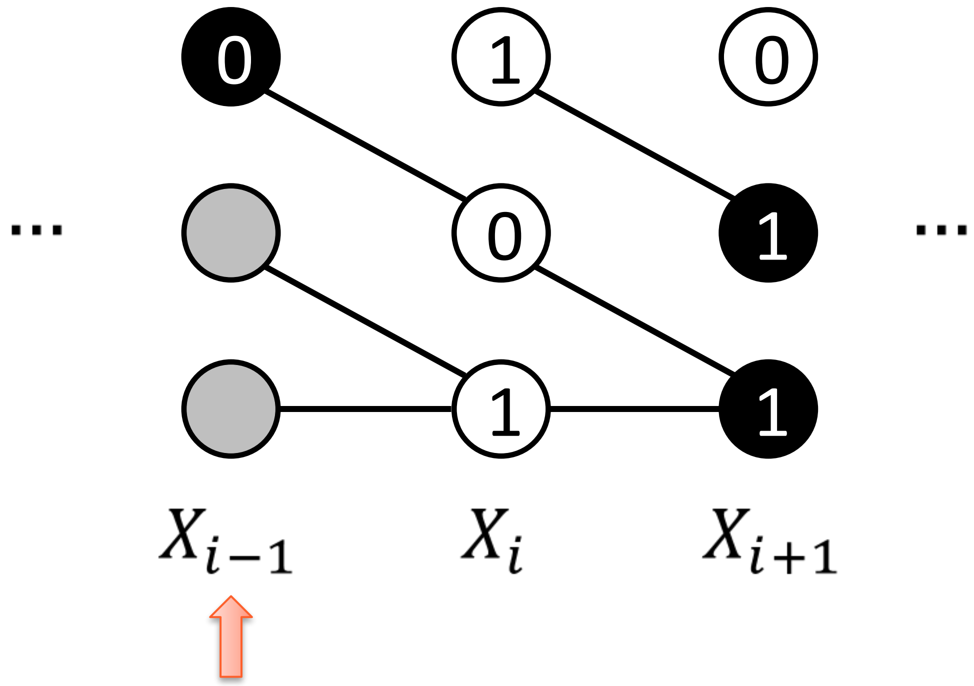

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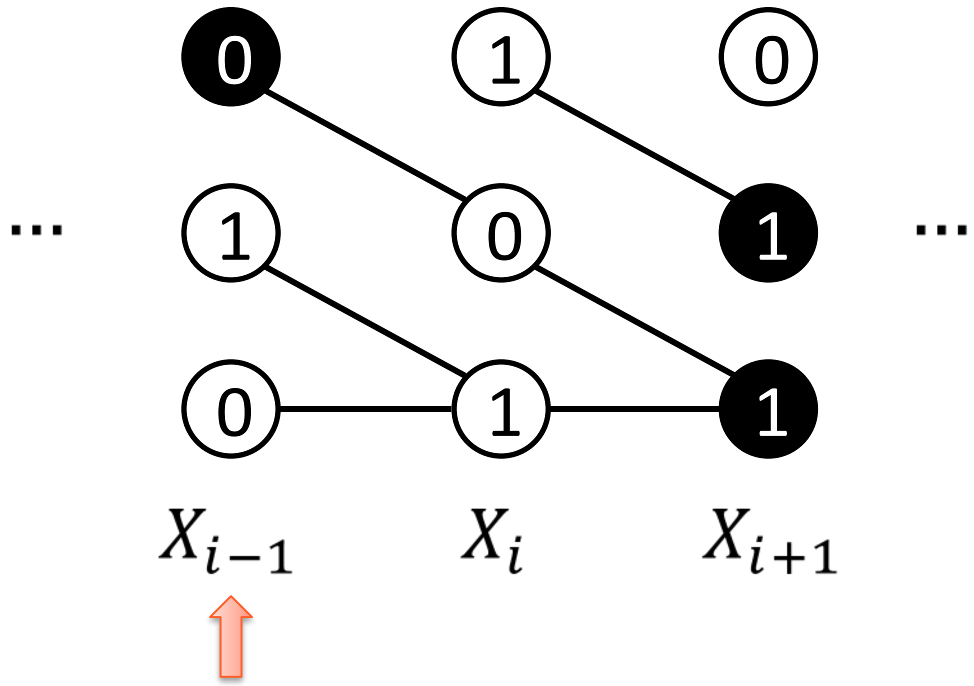

Solver 1: Label Propagation

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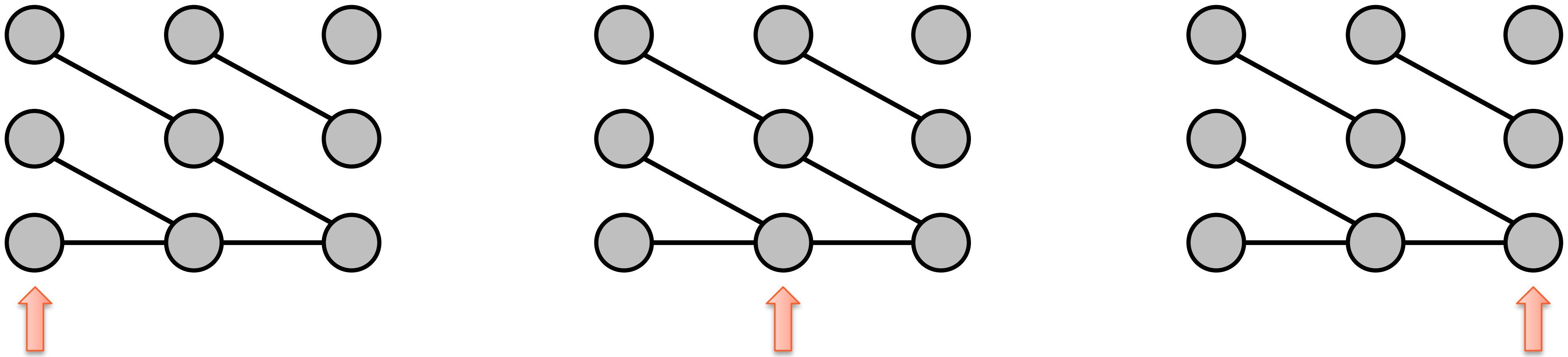
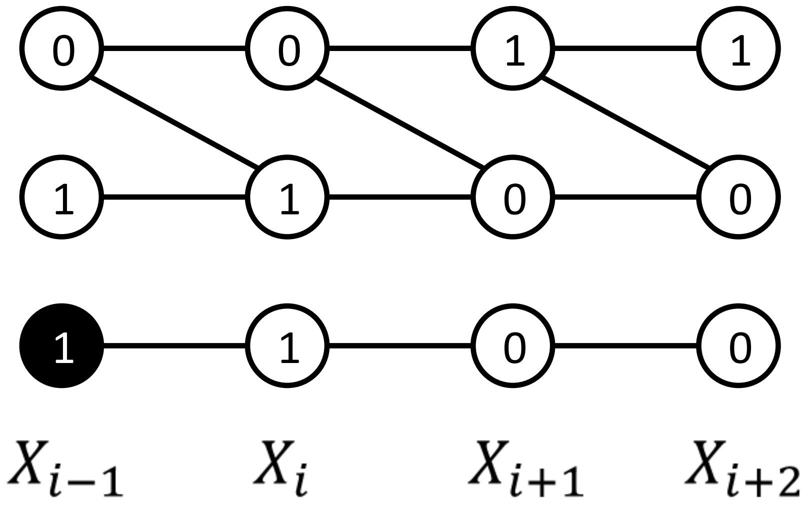
- Propagate labels from one shape to others while avoiding conflicts
  - Among all starting shapes, take the solution with the minimal topology and costs
  - Guarantees to be free of conflicts
  - May not be optimal in topology simplicity or geometric cost



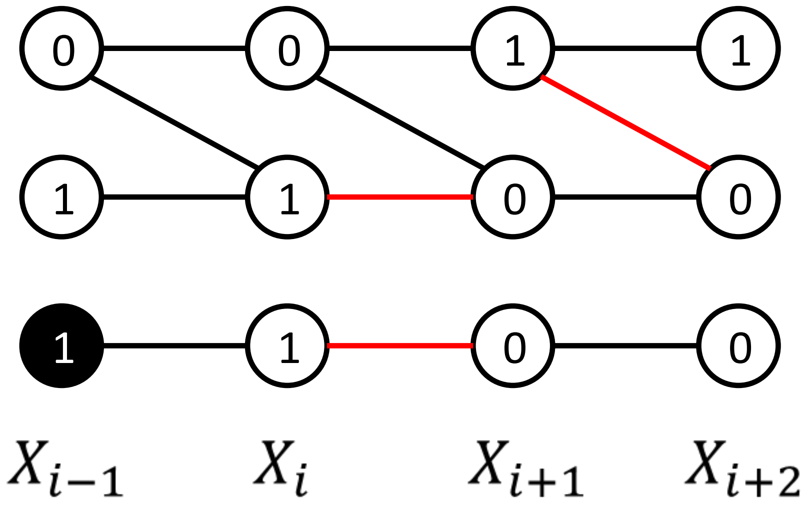
Solver 2: State-space Search

- *State*: a labelling of all candidates, and a set of constrained candidates
  - In a queue sorted by topology + geometric cost



Solver 2: State-space Search

- *State*: a labelling of all candidates, and a set of constrained candidates
  - In a queue sorted by topology + geometric cost
- If the popped state has conflicts:



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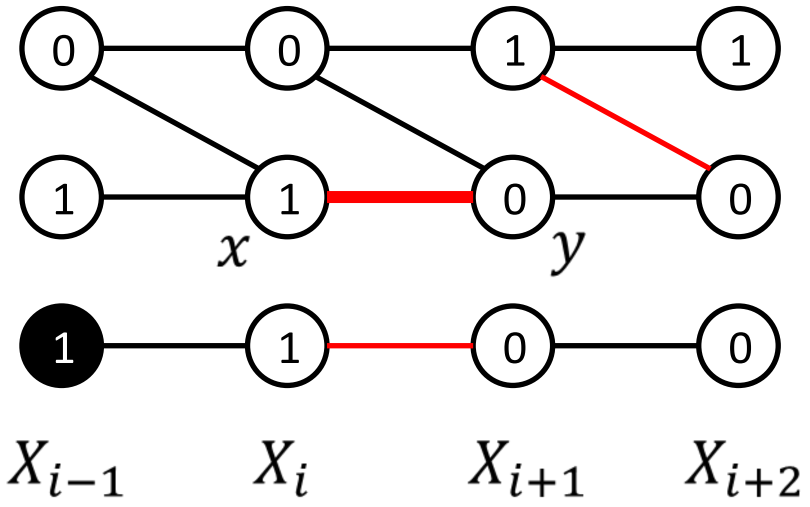
- If the popped state has conflicts:
  - Pick a conflict $\{x \in X_i, y \in X_{i+1}\}$
  - Create 2 new states by either constraining x's label to be 0 or y's label to be 1



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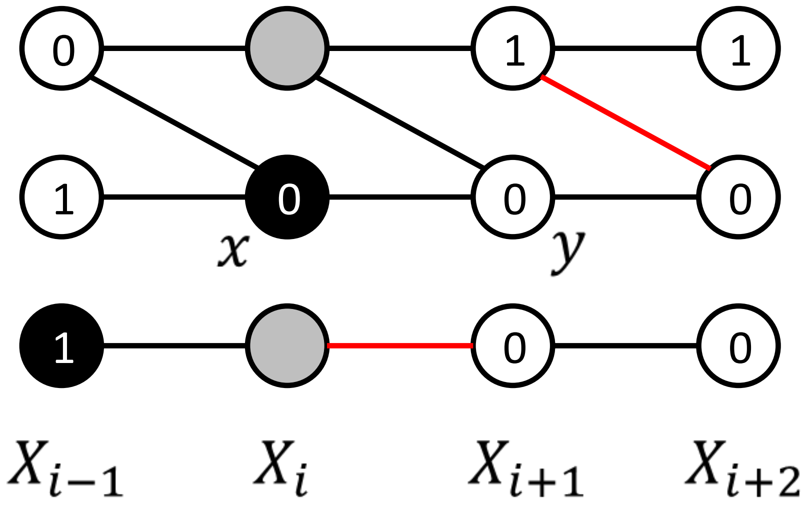

Solver 2: State-space Search

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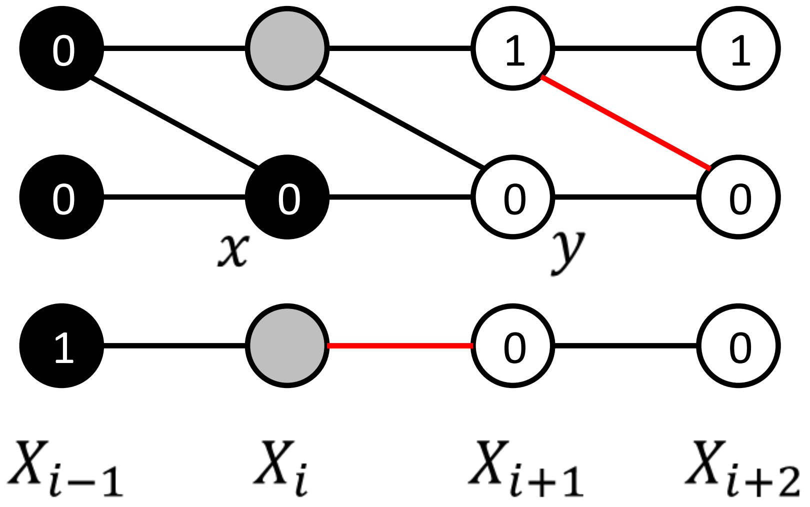

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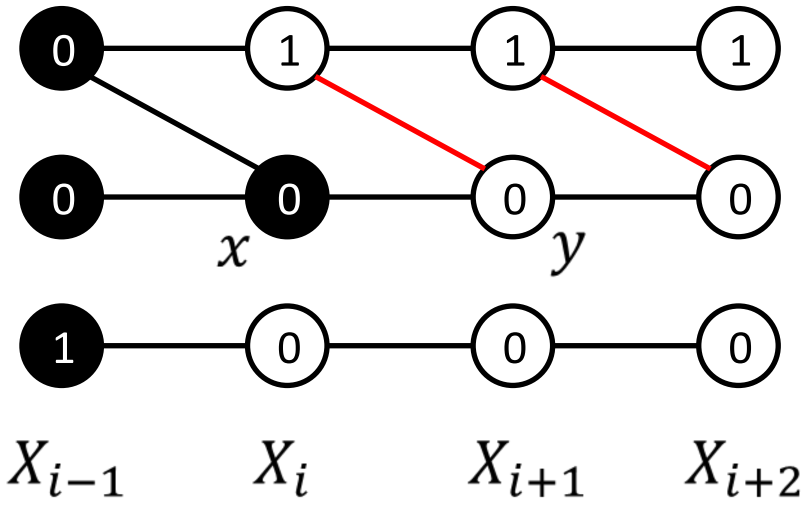
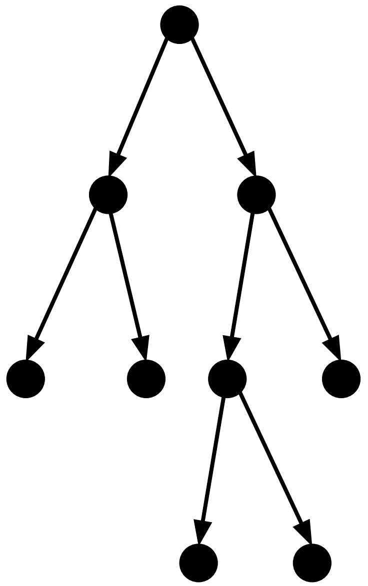
  - In a queue sorted by topology + geometric cost
- If the popped state has conflicts:
  - Pick a conflict $\{x \in X_i, y \in X_{i+1}\}$
  - Create 2 new states by either constraining x's label to be 0 or y's label to be 1
- Terminate otherwise



Solver 2: State-space Search

- Best-first search in a binary tree of states
- Returns optimal conflict-free labelling
  - Assuming [Zeng 20] is optimal
- High computational cost
  - # iterations can be exponential in total # candidates



Solver 3: Beam Search

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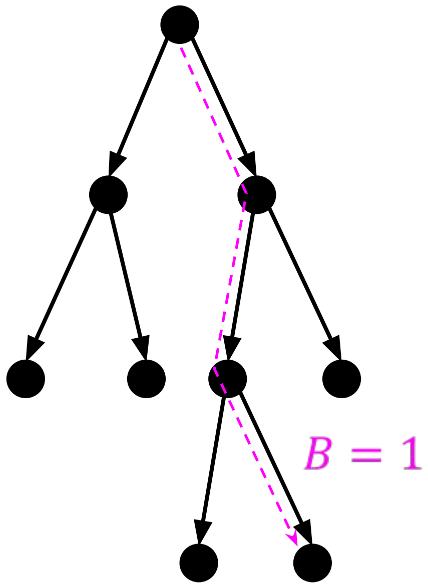
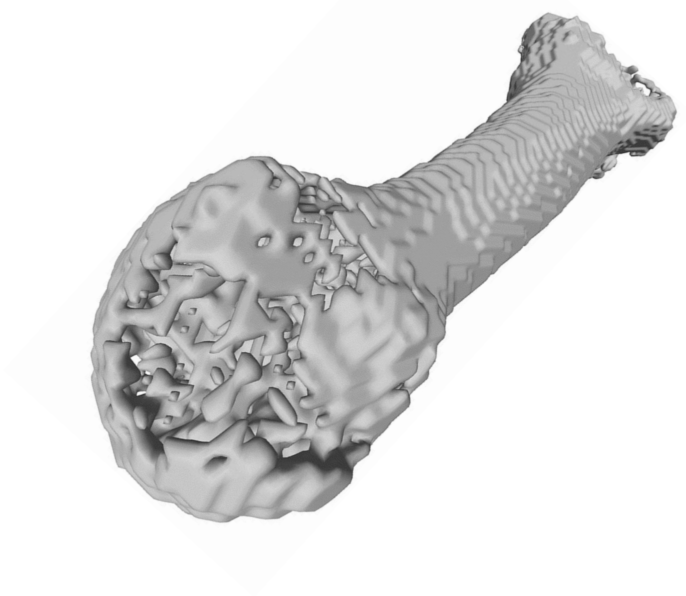
- Limit queue size to a constant B
  - Keep only best B states
- Trade off optimality for efficiency
  - # iterations linear in total # candidates



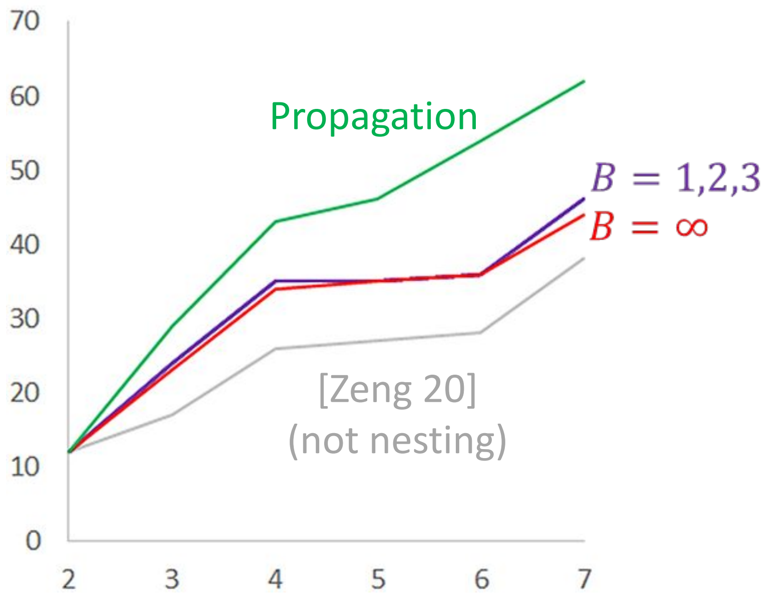
Solver Comparison

Level set of
CT bone scan



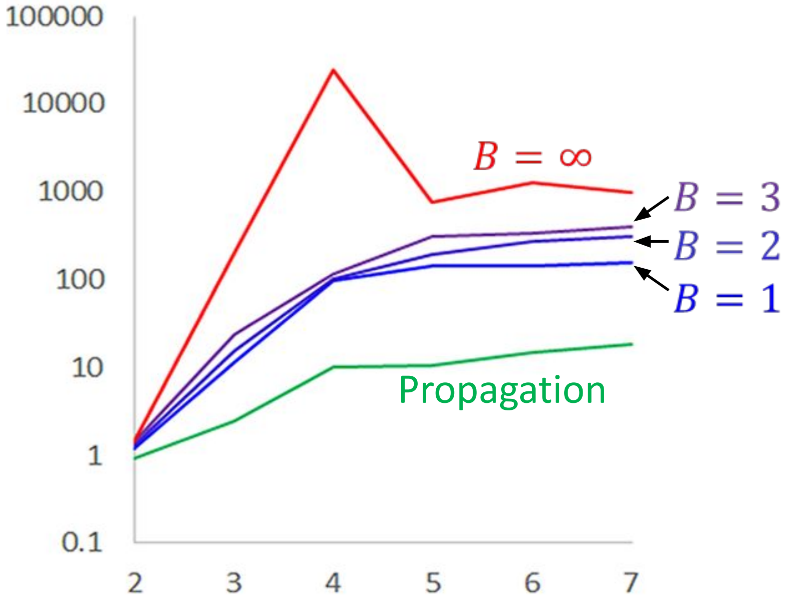
components: 31
handles: 371
voids: 106

Topo. features



Level sets

Running time (secs)



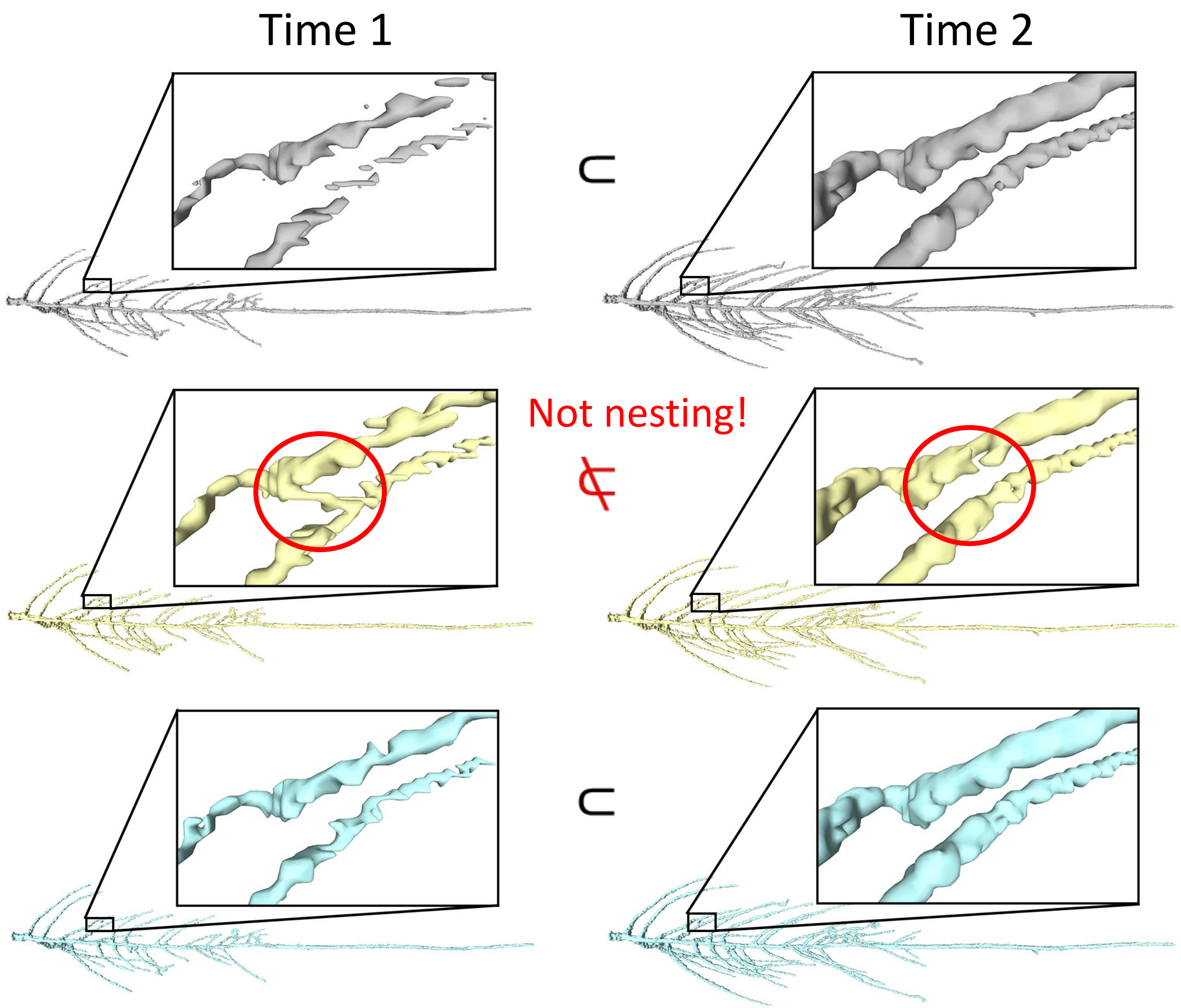
Level sets

Results: Roots

Input:

Single-shape [Zeng 20]:
(Simply connected)

Our method:
(Simply connected)



Results: Roots

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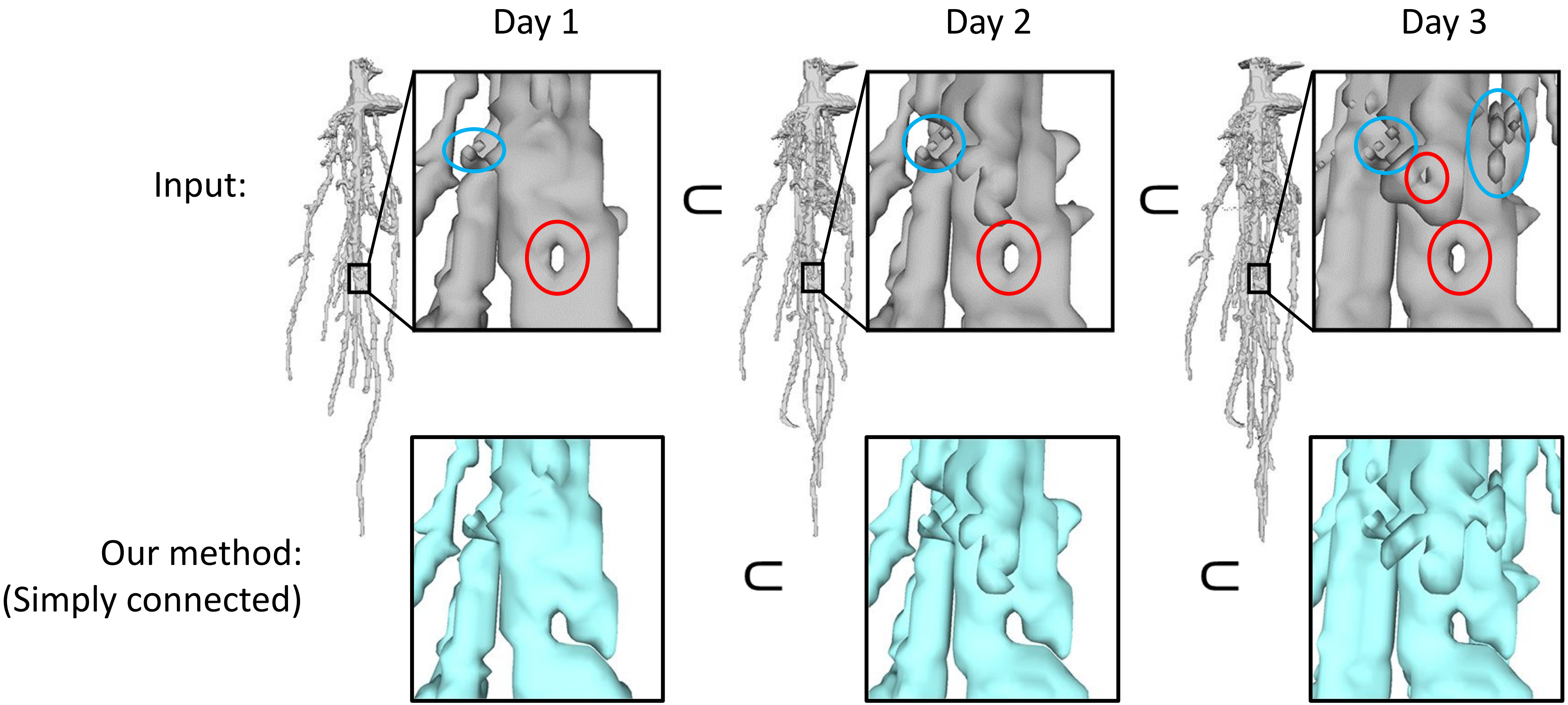

Results: Brain

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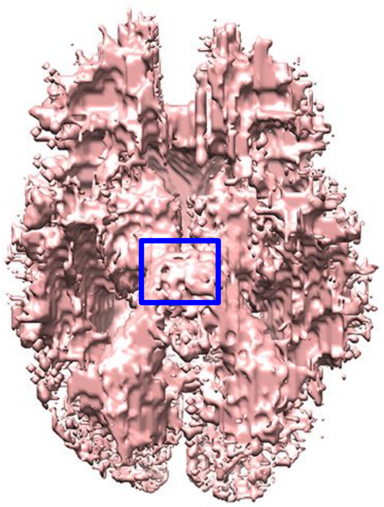
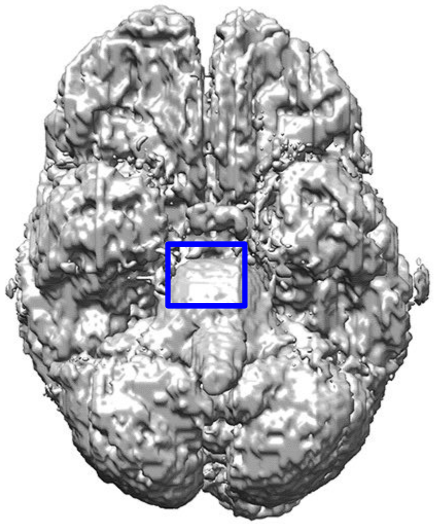
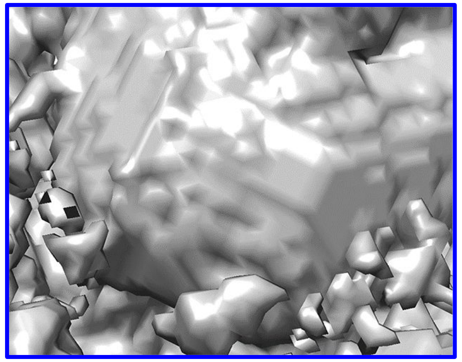
James McKelvey School of Engineering

Cerebrospinal fluid

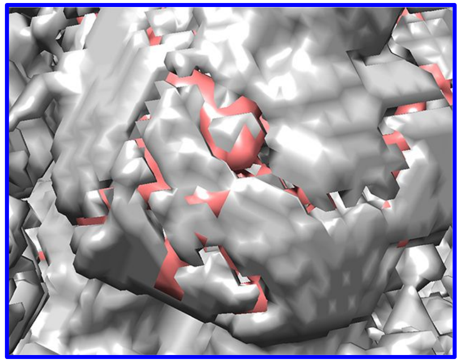


White matter

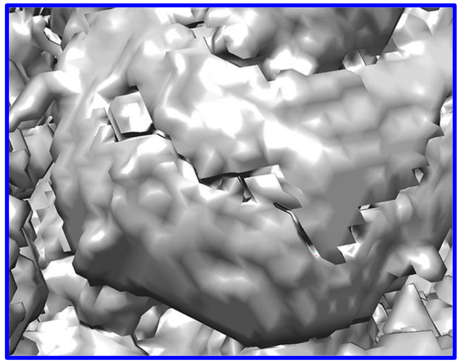




Input (nested)



Single-shape [Zeng 20]
(simply connected; not nested)



Our method
(simply connected and nested)

Limitations and Future Works

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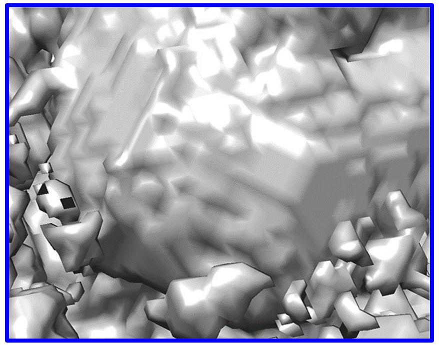
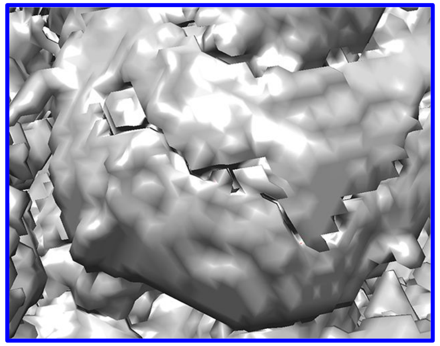
- Need more "natural-looking" candidates and "semantic" geometric costs
- Handling non-cubical complexes
- How to simplify a (not necessarily nesting) shape collection in a consistent way?



Input



Our method

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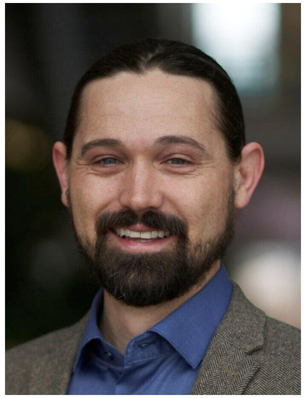
Danforth Plant Science Center



Chris Topp

Dan Zeng